Transformation of Received Signal Polarization Angle to the Plane of the Ecliptic

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Theme

THE solar occultation of the Pioneer VI spacecraft in November 1968 provided the first opportunity to measure the polarization rotation of a CW signal in the solar corona. The received signal polarization angle measured with the Jet Propulsion Laboratory's 64-m Mars station azimuth-elevation mount antenna was transformed to the plane of the ecliptic. The derivation of the equations required for this transformation is presented.

Contents

The November 1968 superior conjunction of the Pioneer VI spacecraft provided the first opportunity to measure the polarization rotation of a CW signal in the solar corona.[1] The polarization angle measurement was made using the Jet Propulsion Laboratory's 64-m Mars station azimuth-elevation (Az-El) mount antenna located at Goldstone, California. Interpretation of the data required that the signal polarization angle measured with the Az-El receiving antenna be transformed to the plane of the ecliptic.[2]

Define unit vectors (Fig. 1)

$$\mathbf{s} = \frac{\mathbf{r} \times \mathbf{r}_p}{|\mathbf{r} \times \mathbf{r}_p|} \tag{1a}$$

and

$$\mathbf{u} = \frac{\mathbf{k}_\varepsilon \times \mathbf{r}_p}{|\mathbf{k}_\varepsilon \times \mathbf{r}_p|} \tag{1b}$$

where $\mathbf{r}$ = vector from Earth center to local station, $\mathbf{r}_p$ = vector from local station to spacecraft probe, and $\mathbf{k}_\varepsilon$ = unit northward vector perpendicular to plane of ecliptic. The vectors **s** and **u** (Fig. 2) are both perpendicular to the station/spacecraft probe line of sight. In addition, **s** is parallel to the plane of the local horizon and **u** is parallel to the plane of the ecliptic.

The signal polarization (nominally perpendicular to the plane of the ecliptic) referred to the plane of the local horizon is

$$\tau = \theta_p - p \tag{2}$$

where τ = signal polarization angle referred to **u** (measured counterclockwise looking toward the probe, Fig. 2), deg, θ_p = signal polarization angle referred to **s** (ground instrumentation measurement defined counterclockwise looking toward the probe relative to the local horizon,[3] (Fig. 2) deg, and p = the angle between the vectors **s** and **u**, both perpendicular to the station-to-probe line of sight (Fig. 2) deg.

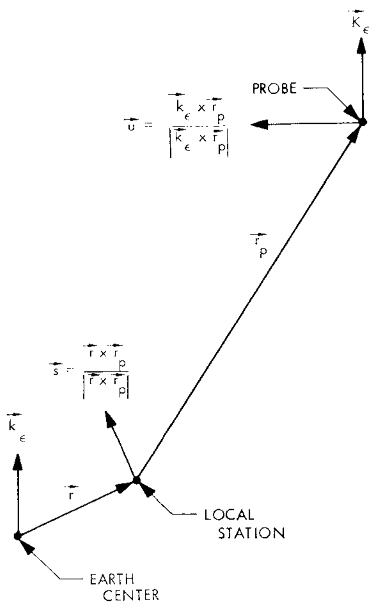

Fig. 1 Vectors used to relate probe polarization in the ecliptic plane to that measured at the local station.

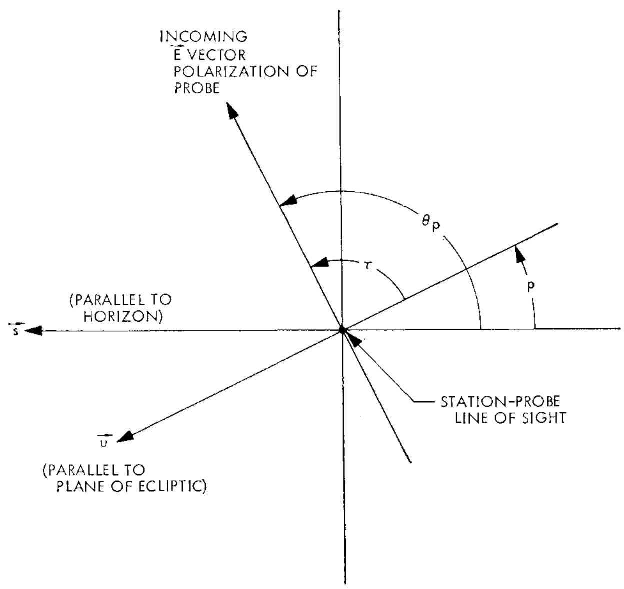

Fig. 2 Polarization angle relationships looking along the station-to-probe line of sight.

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It is not satisfactory to obtain the angle p from the dot product of **s** and **u** since the cosine relationship loses the sign information required when p goes through zero. Define the vector

$$\mathbf{c} = \mathbf{u} \times \mathbf{s} = \mathbf{E} \sin p \tag{3}$$

where

$$\mathbf{E} = \text{unit vector } \mathbf{r}_p / r_p$$

Then, since $\mathbf{u} \times \mathbf{s}$ and **E** are parallel to each other

$$\sin p = (\mathbf{u} \times \mathbf{s})/\mathbf{E} \tag{4}$$

and

$$\sin p = -(k_x \sin\sigma + k_y \cos\sigma)/(a_x{}^2 + a_y{}^2 + a_z{}^2)^{1/2} \tag{5}$$

where[4]

$$a_x = k_y \sin\gamma - k_z \cos\gamma \sin\sigma$$

$$a_y = -k_z \cos\gamma \cos\sigma - k_x \sin\gamma$$

$$a_z = \cos\gamma\,(k_x \sin\sigma + k_y \cos\sigma)$$

$$k_x = -[\sin\phi \cos\theta \sin\gamma(t) \sin\varepsilon + \sin\phi \sin\theta \cos\gamma(t) \sin\varepsilon + \cos\phi \cos\varepsilon]$$

$$k_y = \sin\theta \sin\gamma(t) \sin\varepsilon - \cos\theta \cos\gamma(t) \sin\varepsilon$$

$$k_z = -\cos\phi \cos\theta \sin\gamma(t) \sin\varepsilon - \cos\phi \sin\theta \cos\gamma(t) \sin\varepsilon + \sin\phi \cos\varepsilon$$

θ = local station longitude (referred to Greenwich longitude), deg

ϕ = local station latitude, deg

$\gamma(t)$ = angle between the vernal equinox and Greenwich longitude, deg

ε = angle between the equatorial plane and the plane of the ecliptic, deg

σ, r = local station Az-E1 coordinates to the spacecraft probe, deg

Equation (5) is used with Eq. (2) to refer the measured polarization at the local station to the plane of the ecliptic. These equations have been programmed on the UNIVAC 1108 computer (JPL ID 58790000, also submitted to COSMIC, University of Georgia Computer Center, Athens, Ga.). Computations were made for the duration of the Pioneer VI experiment and were used to compare measured and calculated polarization angles necessary to interpret the data and relate to the solar corona.[2]

The computer program has been adapted to a HA-Dec (hour angle-declination) antenna, using the fact that during meridian transit Az-E1 and HA-DEC antennas are similarly aligned.[5] This feature is presently being used by the NASA/JPL Deep Space Network for polarization predictions.

References

[1] Levy, G. S., et al., "Pioneer VI: Measurement of Transient Faraday Rotation Phenomena Observed During Solar Occultation," *Science*, Vol. 166, 1969, p. 596.

[2] Stelzried, C. T., et al., "The Quasi-Stationary Coronal Magnetic Field and Electron Density as Determined from a Faraday Rotation experiment," *Solar Physics*, Vol. 14, No. 2, Oct. 1970, pp. 440–456.

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[4] Stelzried, C. T., "A Faraday Rotation Measurement of a 13-cm Signal in The Solar Corona," TR 31-1401, July 15, 1970, Jet Propulsion Lab., Pasadena, Calif.

[5] Stelzried, C. T., et al., "Received Signal Polarization Tracking Using an HA-Dec Antenna," Space Programs Summary 33-62, Vol. II, March 31, 1970, Jet Propulsion Lab., Pasadena, Calif.